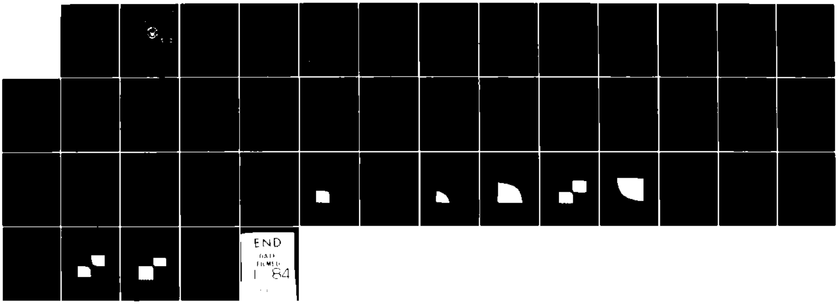
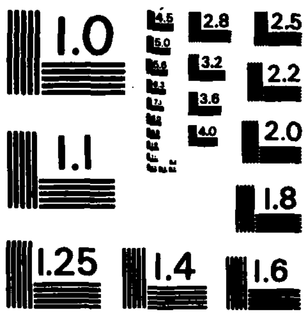
AD-A135 293 EVOLUTION EQUATIONS OF C(3)I: CANNONICAL FORMS AND THEIR PROPERTIES(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA P H MOOSE OCT 83 1/1

UNCLASSIFIED F/G 12/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

A135293

NPS-62-83-059

NAVAL POSTGRADUATE SCHOOL

Monterey, California

EVOLUTION EQUATIONS OF C^3I:
CANNONICAL FORMS AND THEIR PROPERTIES

by

Paul H. Moose

October 1983

Prepared for:
Chief of Naval Research
Arlington, VA 22217

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral John J. Ekelund
Superintendent

D. A. Schrady
Provost

The work reported herein was support by Chief of Naval Research, Arlington, VA 22217.

Prepared by:

Paul H. Moose, Assoc. Professor
Department of Electrical Engineering

Reviewed by:

M. G. Sovereign, Chairman
Command, Control & Communications Academic Group

A. Sheingold, Chairman
Electrical Engineering Department

Released by:

W. M. Tolles
Dean of Research

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS-62-83-059	2. GOVT ACCESSION NO. A135293	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EVOLUTION EQUATIONS OF C^3I: CANNONICAL FORMS AND THEIR PROPERTIES		5. TYPE OF REPORT & PERIOD COVERED Research - 1 Jan 83-1 Aug 83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Paul H. Moose		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 6153N: RR014-02-01 N0001483WR30082
11. CONTROLLING OFFICE NAME AND ADDRESS Chief of Naval Research Arlington, VA 22217		12. REPORT DATE October 1983
		13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper, five different two-species non-linear evolution equations are described that represent, (a) mixed attrition Lanchester combat with resupply and (b) four different models of information war. Their dynamical properties are analyzed and it is shown that four of the five are environmentally unstable. The meaning of this is interpreted for C^3I, counter-C^3, and intelligence operations. For the environmentally unstable systems it is shown that the relationships between the parameters in the model are

20. (Abstract continued)

critical in establishing their mode of behavior, whereas initial conditions are critical in determining the actual trajectories of evolution.

EVOLUTION EQUATIONS OF C^3I:

CANNONICAL FORMS AND THEIR PROPERTIES

Paul H. Moose

In this paper, five different two-species non-linear evolution equations are described that represent, (a) mixed attrition Lanchester combat with resupply and (b) four different models of information war. Their dynamical properties are analyzed and it is shown that four of the five are environmentally unstable. The meaning of this is interpreted for C^3I, counter-C^3I, and intelligence operations. For the environmentally unstable systems it is shown that the relationships between the parameters in the model are critical in establishing their mode of behavior, whereas initial conditions are critical in determining the actual trajectories of evolution.

In several earlier papers, we have suggested the use of non-linear evolution equations to model the dynamics of the average information of each of two opposing sides engaged in conflict. Moose, [1980], developed the properties of one particular version of a model for counter-C^3; counter-C^3 refers to "deliberate acts to distort or deny information or to confuse or deceive one's opposition." In that paper several alternative counter-C^3 models, as well as a model for intelligence activities, were suggested.

In a subsequent paper, Moose, [1983], developed a four-species model to account for interactions between the combat forces and the information on each side. Some of the most general properties of that model exposed the fact that, unlike the earlier counter-C^3 model, a four-species model is environmentally unstable. That is, for some ranges of the parameters of the model, the system is unstable, whereas for others it is stable. By contrast, an environmentally stable model is stable for all values of its parameters. (Passive electrical and mechanical networks are familiar forms of environmentally stable systems, whereas automatic control systems are environmentally unstable systems.)

Evolution equations have been used to model phenomena in the social sciences before. Lanchester, [1916], initiated the use of coupled first-order differential equations to model the attrition of military forces engaged in combat. Richarson, [1960] attempted to fit the growth of military arms in "arms races" to linear evolution models. In a recent survey article, Intriligator, [1982], cited eleven examples of the use of differential equations and systems of differential equations to account for various political phenomena related to conflict resolution.

Non-linear evolution equations are particularly interesting because they have more than one stationary point. This enables us to duplicate mathematically the unusual behavior of many complex phenomena that seem to be attracted for a period of time toward a particular condition, then in an apparently arbitrary and unpredictable way change to another characteristic condition. This branch of mathematics is now referred to as chaos theory and the characteristic types of system behavior are called "strange attractors."

In this paper, we shall study the properties of five different two-species systems. One of these is a Lanchester type combat force attrition model with a mixture of area and directed fire; the relative mix will be made dependent on parameters which we presume are related to the "goodness" of the C^3I system

of each side. In addition we shall study three counter-C^3 models, including the model of Moose, [1980]. These models are intended to capture the essence of "information war." The fifth model was suggested by Moose [1980] to describe the struggle between two sides for "intelligence information." These five models are shown to be of three cannonical types. The stationary points and stability of each type are derived. Types "one" and "three" are environmentally unstable and, as shown by example, are very rich in the nature of their behavior. Type "two" is environmentally stable.

The Equations and Their Interpretation

Coupled two-species evolution equations are of the general form

$$\left.\begin{aligned} \dot{X} &= -F_x(X, Y, \underset{\sim}{\mu}) + V_x \\ \dot{Y} &= -F_y(X, Y, \underset{\sim}{\mu}) + V_y \end{aligned}\right\} \qquad (1)$$

F_x and F_y are attrition functions. They depend on the state variables X and Y and on a set of parameters $\underset{\sim}{\mu}$. V_x and V_y are replenishment terms. They, as well as the μ may be constants or time dependent. If the parameters and replenishments are constants, the system is said to be autonomous. We shall consider only autonomous systems.

Mixed attrition of combat forces is characterized by;

1) A rate of loss due to directed fire which is proportional to the size of the opposition's force.
2) A rate of loss due to area fire which depends on the size of the opposition's force and of the density of one's own force.
3) A natural rate of loss due to accident, illness, disertion, etc.
4) Reinforcement and resupply of the combat force in the field.

In order to account for these conditions, let

$$\left.\begin{aligned} \dot{X} &= -b_xX - \alpha_y \mu_y Y - \alpha_y(1 - \mu_y)XY + V_x \\ \dot{Y} &= -b_yY - \alpha_x \mu_x X - \alpha_x(1 - \mu_x)XY + V_y \end{aligned}\right\} \qquad (2)$$

be the specific characterization of (1). The parameters b_x and b_y are the natural loss rates, α_y and α_x are related to the rates of fire and probability of kill of the weapons systems in use, and μ_y and μ_x describe the fraction, of Y's and X's forces respectively, that are aimed or directed fire. If a μ is zero, then all the fire is area fire. If a μ is one, then all the

fire is aimed. The V_x and V_y are the average rates of reinforcement and resupply of X and Y.

We shall be interested in analyzing the behavior of (2) in the two-dimensional parameter space μ_x, μ_y. There are two interpretations we can make for the μ's. In terms of C^3I, we argue that μ's monotonically increases toward one as the C^3I is improved, i.e., better C^3I improves targeting on the average and hence the fire tends more toward aimed fire and less toward area fire. Another interpretation is that of an exposed force attacking a concealed defender. Suppose X is attacking and Y is defending. Since X exposes his position during attack, Y may aim his fire and μ_y tends to one. On the other hand, X must fire into Y's defended area, not knowing exactly where Y is, so μ_x tends toward zero. This is similar to a case studied by Brackney, [1959].

In an information war, the state variables X and Y represent the average information each side possesses about the combat environment. We are interpreting information here in its theoretical sense as lack of uncertainty. There are many microscopic informational entities that make up the state of the combat environment. These include your own and the enemies force locations, capabilities and even his intentions. The true state of the environment is never known exactly and moreover, without continuing intelligence and combat reports to update or replenish the information, it will decay over time due to the dynamic nature of the environment.

Each information war model will contain

1) A natural rate of loss due to the dynamic properties of the environment.
2) Replenishment of information from intelligence, sensor, combat and other reports.

3) A rate of loss due to deliberate acts of the opposition to deceive, confuse, jam, etc. This is the counter-C^3 term and it is in this term that the three information war models differ in their assumptions.

$$\dot{X} = -b_x X - \rho_y (X_o - X)Y + V_x$$
$$\dot{Y} = -b_y Y - \rho_x (Y_o - Y)X + V_y \tag{3a}$$

$$\dot{X} = -b_x X - \alpha_y Y + V_x$$
$$\dot{Y} = -b_y Y - \alpha_x X + V_y \tag{3b}$$

$$\dot{X} = -b_x X - \mu_y XY + V_x$$
$$\dot{Y} = -b_y Y - \mu_x XY + V_y \tag{3c}$$

In all three models b_x and b_y represent natural loss rates and V_x and V_y represent replenishment rates for X and Y respectively.

The assumption in (3a) is that counter-C^3 effectiveness of Y against X depends on the product of Y's knowledge (Y) and X's ignorance (X_o - X). (X_o is the maximum possible level of information of X.) The premise is that counter-C^3 is more effective against an already confused or ignorant enemy than against an intelligent one. Furthermore, that one is more effective in devising counter-C^3 the better informed he is about the environment. ρ_y and ρ_x are parameters.

In (3b), the assumption is that the enemies' lack of knowledge is immaterial in matters of counter-C^3. Its effectiveness only depends on one's own knowledge of the environment. α_y and α_x are parameters, here assumed to be positive. We note in passing that for negative values of α_x and α_y, (3b) are Richarson's, [1960] linear equations for arms races.

In (3c) the assumption is that an ignorant or already confused enemy cannot be further confused by counter-C^3 acts, but that in fact, the effectiveness of such tactics actually increases with his as well as one's own accurate state of knowledge about the environment. μ_y and μ_x are parameters, presumed to be positive but possibly greater than one. Eq.'s (3c) were those analyzed for equilibria and stability in the paper by Moose, [1980] and found to be environmentally stable.

The final model we wish to consider is in the area of intelligence information acquisition. Like the other information models X and Y represent the average values of correct information about an uncooperative environment. The properties to be modeled are:

1) A natural rate of information loss due to the dynamic environment.

2) Replenishment of information through normal open intelligence gathering channels.

3) Information growth through uncooperative channels that "leak" more information about the enemy in proportion to his ignorance and are better "exploited" by one's own assets in proportion to one's own knowledge. The latter is to account for more effective targeting of limited acquisition resources.

$$\begin{aligned} \dot{X} &= -b_x X + \mu_x (Y_o - Y) X + V_x \\ \dot{Y} &= -b_y Y + \mu_y (X_o - X) Y + V_y \end{aligned} \tag{4}$$

Eq.'s (4) are the intelligence information evolution equations that account for these proposed properties. μ_x and μ_y are parameters. They are presumed to be positive but we could imagine that they are negative if the "leaks" are in fact "misinformation" being used for the purpose of deception.

In summary, the five systems of equations represent models of;

Eq. (2) : Mixed attrition combat,

Eq. (3a) : Information war; counter-C^3 depends on one's own knowledge and opposition's ignorance,

Eq. (3b) : Information war; counter-C^3 depends on one's own knowledge,

Eq. (3c) : Information war; counter-C^3 depends on the product of one's own knowledge and opposition's knowledge,

Eq. (4) : Intelligence information; intelligence grows in proportion to enemy ignorance and one's own knowledge.

Cannonical Forms

Eq.'s (2), (3a) and (3b) are of a cannonical form we shall call "Type 1" equations. Type 1 equations are designated in accordance with the notation of Eq. (2), as

$$
\begin{aligned}
\dot{X} &= -b_x X - \alpha_y \mu_y Y - \alpha_y (1 - \mu_y) XY + V_x \\
\dot{Y} &= -b_y Y - \alpha_x \mu_x X - \alpha_x (1 - \mu_x) XY + V_y
\end{aligned}
\qquad (5)
$$

The parameters b_x and b_y are positive and have the interpretations of natural loss rates. Eqs. (2) are obtained from (5) by restricting α_x and α_y to be positive and μ_x and μ_y to be greater than or equal to zero and less than or equal to one. Eq. (3b) is obtained from (5) by setting μ_x and μ_y equal to one. Eqs. (5) may be obtained from (3a) by the transformations

$$
\left.
\begin{aligned}
\mu_y &= \frac{X_0}{X_0 - 1} \quad , \quad \mu_x = \frac{Y_0}{Y_0 - 1} \\
\alpha_y &= \rho_y (X_0 - 1) \quad , \quad \alpha_x = \rho_x (Y_0 - 1)
\end{aligned}
\right\}
\qquad (6)
$$

Since, as we shall explain shortly, X_0 and Y_0 are restricted to being greater than one, μ_y and μ_x will be greater than one and α_y and α_x will be positive in this transformation.

In summary, Eq. (5) in a cannonical form for Eq. (2), the mixed attrition model, Eq. (3a) the counter-C^3 model depending on the enemies' ignorance for deceptive successes, and Eq. (3b), the counter-C^3 model that is independent of the enemies' state of knowledge. The range of the parameter pair μ_x, μ_y determines the model to which (5) applies. Between zero and one, we have the mixed attrition model. For values greater than or equal to one we have two of the counter-C^3 models.

Eqs. (3c) are of a unique form which we shall designate "Type 2" equations. Again, we shall investigate their properties in the two-dimensional parameter space μ_x, μ_y . Eqs. (4), the intelligence equations are also different and we shall designate them as "Type 3" equations.

Stationary Points

In general, two-species, quadratic evolution equations may have four stationary points, that is points in the X-Y plane for which $\dot{X}$ and $\dot{Y}$ are simultaneously zero. However, because there are no X^2 or Y^2 terms these equations all have two stationary, or equilibrium, points. In all of these models, the state variables, X and Y only have intelligent interpretation when they are non-negative. Furthermore, the purpose of the replenishment terms is to make up for attrition in order to maintain their values at a desired level. We shall thus presume V_x and V_y are picked to establish one of the equilibrium points in the positive quadrant of the state space. It is mathematically convenient to pick this point as $\left\{X_{e1}, Y_{e1}\right\} = \left\{1,1\right\}$, by suitable scale changes if necessary. Hereafter we shall refer to this as the unity equilibrium point. This prescribes a fixed relationship between the model parameters and the replenishment terms. Namely;

Type 1:

$$\begin{aligned} V_x &= b_x + \alpha_y \\ V_y &= b_y + \alpha_x \end{aligned} \tag{7}$$

Type 2:

$$V_x = b_x + \mu_y \qquad (8)$$
$$V_y = b_y + \mu_x$$

Type 3:

$$V_x = b_x - \mu_x(Y_o - 1) \qquad (9)$$
$$V_y = b_y - \mu_y(X_o - 1).$$

It is easily seen that (7), (8) and (9) substituted into (5), (3c) and (4) along with X = Y = 1 make $\dot{X}$ and $\dot{Y}$ simultaneously zero for the Type 1, Type 2 and Type 3 equations respectively. Given that unity is an equilibrium point, one may find the other equilibrium point by direct algebraic manipulation. Let it be designated $\{X_{e2}, Y_{e2}\}$. Then

Type 1:

$$\left.\begin{aligned} X_{e2} &= \frac{(b_y/\alpha_x)(b_x/\alpha_y + 1) - \mu_y(b_y/\alpha_x + 1)}{\mu_x(1 - \mu_y) - (b_x/\alpha_y)(1 - \mu_x)} \\ Y_{e2} &= \frac{(b_x/\alpha_y)(b_y/\alpha_x + 1) - \mu_x(b_x/\alpha_y + 1)}{\mu_y(1 - \mu_x) - (b_y/\alpha_x)(1 - \mu_y)} \end{aligned}\right\} \qquad (10)$$

Type 2:

$$\left.\begin{aligned} X_{e2} &= - \frac{b_y}{\mu_x} \ (1 + \mu_y/b_x) \\ Y_{e2} &= - \frac{b_x}{\mu_y} \ (1 + \mu_x/b_y) \end{aligned}\right\} \qquad (11)$$

Type 3:

$$
\left.\begin{aligned}
X_{e2} &= \left[X_o - \frac{b_y}{\mu_y}\right]\left[1 - \frac{1}{Y_o - \frac{b_x}{\mu_x}}\right] \\
Y_{e2} &= \left[Y_o - \frac{b_x}{\mu_x}\right]\left[1 - \frac{1}{X_o - \frac{b_y}{\mu_y}}\right]
\end{aligned}\right\} \qquad (12)
$$

give the locations of the second stationary point for each of the three canonical forms in terms of the model parameters.

We can make some general observations about the locus of these points. For Type 1 systems, the second equilibrium point may lie in any quadrant of the state space. However, if either μ_x or μ_y is greater than one, it cannot be in the third quadrant, i.e., X_{e2} and Y_{e2} cannot both be negative. And for $\mu_x > 1 + b_y/\alpha_x$ and $\mu_y > 1 + b_x/\alpha_y$, it must lie in the first quadrant, i.e., both X_{e2} and Y_{e2} will be positive.

For (3a), the first counter-C^3 model, this requirement corresponds to $\rho_x > b_y$ and $\rho_y > b_x$ according to the transformation required by (6). We may in fact use (6) to define the second equilibrium point of Type 1 equations directly in terms of $\left\{X_o, Y_o, \rho_x, \rho_y\right\}$, the counter-$C^3$ parameters of (3a), by the equivalent form.

Type 1 (alternate form):

$$
\left.\begin{aligned}
X_{e2} &= \frac{X_o(Y_o - 1) + (b_y/\rho_x)(1 - b_x/\rho_y)}{Y_o - b_x/\rho_y} \\
Y_{e2} &= \frac{Y_o(X_o - 1) + b_x/\rho_y(1 - b_y/\rho_x)}{X_o - b_y/\rho_x}
\end{aligned}\right\} \qquad (10a)
$$

For large values of ρ_x and ρ_y, $\left\{X_{e2}, Y_{e2}\right\} \to \left\{X_o \frac{Y_o - 1}{Y_o}, Y_o \frac{X_o - 1}{X_o}\right\}$. For small ρ_x and ρ_y, the equilibrium point moves toward plus infinity until at zero the equations are linear for which there is just one equilibrium point, unity. In between ranges of ρ_x and ρ_y can move the equilibrium point into any quadrant with the behavior quite remarkable near the singular points.

For example, as $\rho_y \to b_x$, $X_{e2} \to -\infty$ but $Y_{e2} \to Y_o$ and vice versa. However, if $\rho_y \to b_x$ and $\rho_x \to b_y$ simultaneoulsy, then $X_{e2} \to X_o$ and $Y_{e2} \to Y_o$. It is for values of ρ_x and ρ_y near these singular points that we find $\left\{X_{e2}, Y_{e2}\right\}$ outside of the first quadrant.

In (10), the original form, for μ_x and μ_y much less than one, a likely range for (2) the mixed attrition Lanchester equations, the second equilibrium point will lie in the fourth quadrant, and be infeasible. For $\mu_x = \mu_y = 1$, the case of (3b), the equations are linear and there is only one equilibrium point, unity. The locus of $\left\{X_{e2}, Y_{e2}\right\}$ is shown in Figure 1 for several values of the model parameters, that might correspond to (2) and in Figure 2 for ranges that might correspond to (3a). It is worth mentioning here that X_o and Y_o are the maximum possible levels of information X and Y can possibly possess. If unity is to be an equilibrium point that is feasible, then X_o and Y_o must be greater than one, a condition we have previously assumed.

For Type 2 systems, X_{e2} and Y_{e2} are always negative. Thus, only one equilibrium point is in the accessible region of the state space. The negative point is not feasible since X and Y cannot be negative quantities. Type 2 equations correspond to the counter-C^3 model of (3c) which postulates counter-C^3 dependent on the product of $\underline{X}$'s and $\underline{Y}$'s knowledge. This was the model analyzed by Moose, [1980] where the same condition was described.

The second equilibrium point for Type 3 system lies in the third quadrant providing V_x and V_y, the constant rates for replacing lost intelligence information, are positive. Even if V_x and V_y are zero; $\{X_{e2}, Y_{e2}\} = \{0, 0\}$.

To summarize: When their first equilibrium point is placed at unity, in the first quadrant of the state space, the second equilibrium point for Type 2 and Type 3 systems is strictly negative, i.e., in the third quadrant of the state space. The second equilibrium point of Type 1 systems may be in any quadrant. However, for sufficiently large values of μ_x and μ_y, or ρ_x and ρ_y, it must be in the first quadrant. Only the first quadrant of the state space is considered a feasible region for the state variables.

Stability

The parameters of evolution equations determine their stability near equilibrium points as well as the location of the equilibrium points. Stability near an equilibrium point, called neighborhood stability, refers to the dynamic property of the system to return to or diverge from equilibrium if perturbed from the equilibrium position.

In order to investigate this, we replace the state variables according to

$$\begin{aligned} X &= \delta_x + X_e \\ Y &= \delta_y + Y_e \end{aligned} \tag{13}$$

and expand (1) about $\{X_e, Y_e\}$ in a Taylor series. Retaining only the first order terms in the expansion, which is valid for small δ_x and δ_y, one obtains the linear state equation

$$\begin{bmatrix} \dot{\delta}_x \\ \dot{\delta}_y \end{bmatrix} = \underset{\sim}{C}_e \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} \tag{14}$$

where $\underset{\sim}{C}_e$, called the conflict matrix, is

$$\underset{\sim}{C}_e = \begin{bmatrix} -\frac{\partial F_x}{\partial x} & -\frac{\partial F_x}{\partial y} \\ -\frac{\partial F_y}{\partial x} & -\frac{\partial F_y}{\partial y} \end{bmatrix} \tag{15}$$

and each of the partial derivations are to be evaluated at the equilibrium point under investigation. [See Moose, [1983] or May, [1974] for a more complete description of the perturbation method for this class of non-linear evolution equations.]

The conflict matrix determines the stability or instability of the system. If all its eigenvalues are in the left half plane, the system is "stable"; i.e., perturbations will die out. If one or both of its eigenvalues are in the right half plane, a minor perturbation will grow and the system will diverge from that equilibrium. In this case it is said to be "unstable." If its eigenvalues are complex, (they must be complex conjugates if they are), the system dynamics will be oscillating. The oscillations will die out if it is stable, but will grow if it is unstable. If the eigenvalues are in the left half plane for all possible values of the parameters, the system is said to be "environmentally stable." If for some values of the parameters, the system is stable but for others it is unstable, the system is said to "environmentally unstable."

The conflict matrices for the three cannonical forms are;

<u>Type 1</u>:

$$\underset{\sim}{C}_e = \begin{bmatrix} -b_x - \alpha_y(1-\mu_y)Y_e & -\alpha_y\mu_y - \alpha_y(1-\mu_y)X_e \\ -\alpha_x\mu_x - \alpha_x(1-\mu_x)Y_e & -b_y - \alpha_x(1-\mu_x)X_e \end{bmatrix} \tag{16}$$

Type 1 (alternate form):

$$\underset{\sim}{C}_e = \begin{bmatrix} -b_x + \rho_y Y_e & -\rho_y(X_o - X_e) \\ -\rho_x(Y_o - Y_e) & -b_y + \rho_x X_e \end{bmatrix} \tag{16a}$$

Type 2

$$\underset{\sim}{C}_e = \begin{bmatrix} -b_x - \mu_y Y_e & -\mu_y X_e \\ -\mu_x Y_e & -b_y - \mu_x X_e \end{bmatrix} \tag{17}$$

Type 3

$$\underset{\sim}{C}_e = \begin{bmatrix} -b_x + \mu_x(Y_o - Y_e) & -\mu_x X_e \\ -\mu_y \mu_e & -b_y + \mu_y(X_o - X_e) \end{bmatrix} \quad (18$$

Equations (16) through (18) show how the conflict matrix and hence system stability depends on the parameters of each of the cannonical types. Recall that $\left\{X_{e1}, Y_{e1}\right\} = \left\{1, 1\right\}$ is an equilibrium point located in the feasible region of the state space. In order to obtain the conflict matrices near this point, we simply put $X_e = Y_e = 1$ in (16) through (18), and investigate the properties of the eigenvalues. However, $\left\{X_{e2}, Y_{e2}\right\}$ is also an equilibrium point and we must substitute the appropriate equations from the previous section into (16) through (18) in order to determine stability there.

If equilibrium point two as well as the unity equilibrium is in the feasible region, as is frequently the case in Type 1 systems, then there is the possibility of two feasible stable steady states. Or one or the other may be stable and the other unstable, in which case the system will seek the stable solution. For Type 2 and 3 systems, the stability of the unity point is most

important because the other point is not feasible. However, if the unity point is unstable, and the unfeasible point is stable, then we can forecast that both the state variables will die out, that is, tend toward negative values, instead of growing without bound.

The eigenvalues of $\underset{\sim}{C}_e$ are the roots of the characteristic polynomial

$$D(p) = p^2 - (C_{11} + C_{22})p + (C_{11}\, C_{22} - C_{12}\, C_{21}) \tag{19}$$

where the coefficients are the elements of $\underset{\sim}{C}_e$.

The roots of (19)

$$p_{1,2} = \frac{C_{11} + C_{22}}{2} \pm \left[\left(\frac{C_{11} - C_{22}}{2}\right)^2 + C_{12}\, C_{21}\right]^{1/2} \tag{20}$$

are functions of the system parameters according to (16) through (18). We note that if

$$\left.\begin{aligned} &(C_{11} + C_{22}) < 0 \\ \text{and} \quad &C_{11}\, C_{22} > C_{12}\, C_{21} \end{aligned}\right\} \tag{21}$$

then $R_e[p_1]$ and $R_e[p_2]$ are less than zero, i.e., p_1 and p_2 are in the left half plane, and the system is stable. Eq. (21), which is called the Routh Test [see, e.g., Truxall, [1972]] is a necessary and sufficient condition for stability. We also note from (20) that $C_{12}\, C_{21} > 0$ is a sufficient condition for both of the roots to be real.

Stability of Type 1 Systems at Unity Equilibrium

Type 1 systems are stable at unity equilibrium for all μ_x and μ_y less than one when

$$[\mu_x - (b_y/\alpha_x + 1)]\, [\mu_y - (b_x/\alpha_y + 1)] > 1. \tag{22}$$

Eq. (22) is the second condition of (21), since the first is always satisfied. The stable and unstable regions determined by (22) are illustrated in Figure 3. Note that when natural loss rates are equal to or greater than the external attrition coefficients, $b_x/\alpha_y > 1$ and $b_y/\alpha_x > 1$, the system is stable for all positive values of μ_x and μ_y less than one. On the other hand, if b_x/α_y and b_y/α_x become very small, then the unity equilibrium point is unstable for all values of μ_x greater than b_x/α_y and μ_y greater than b_y/α_x. In Figures 3a, 3b and 3c, we have sketched the left side of (22) for three specific values of b_x/α_y and b_y/α_x in order to illustrate this point graphically.

The quantities b/α are related to the efficiency of warfare. When warfare is very efficient b/α is small for both sides and war is fundamentally unstable at its unity equilibrium point; that is, the force levels cannot stay at their planned military levels. What happens with larger values for μ, corresponding to greater C^3 efficiency is that it is unstable for relatively larger values of b/α corresponding to lower levels of warfare efficiency.

Since the combat force levels cannot remain at their planned levels, when the system is unstable, we must ask, what will the system trajectory be? This brings the stability properties of the second equilibrium point into play. Because X_{e2}, Y_{e2}, which depend on the system parameters, appear explicitly in the conflict matrix, its eigenvalues are much more complicated functions of the parameters than the eigenvalues at unity equilibrium. A discussion of these properties will be deferred to a separate paper.

For μ_x and μ_y greater than one, Type 1 equations represent the counter-C^3 model of information war of (3a) and we can determine the

conditions from (16a) and (21) for stability to be

$$\left.\begin{array}{l} \rho_x + \rho_y < b_x + b_y \\ \text{and} \quad \left(\dfrac{b_y}{\rho_x} - 1\right)\left(\dfrac{b_x}{\rho_y} - 1\right) > (X_o - 1)(Y_o - 1) \end{array}\right\} \qquad (23)$$

It is easy to show that the second condition implies the first so that the second condition is both necessary and sufficient for the system to be stable. As shown in Figures 4a and 4b, this occurs for values of ρ_x less than b_y and ρ_y less than b_x. As with the Lanchester model of mixed attrition, this information war model becomes unstable at unity equilibrium when the counter-C^3 effectiveness $\{\rho_x, \rho_y\}$ exceeds the natural loss rates, $\{b_x, b_y\}$.

Examples of stable and unstable phase trajectories are shown in Figures 5a, 5b, 5c and 5d for Type 1 systems. Type 1 systems are environmentally unstable systems as defined earlier.

Finally, we note that

$$C_{12}\, C_{21} = \alpha_x\, \alpha_y = \rho_x\, \rho_y (X_o - 1)(Y_o - 1) > 0 \qquad (24)$$

for Type 1 systems at unity equilibrium. Therefore the eigenvalues are real numbers and the trajectories are non-oscillating near there.

Stability of Type 1 Systems for $\mu_x = \mu_y = 1$

This is the case for counter-C^3 equations (3b) or, if we permit α_x and α_y to be negative, the Richarson arms race equations. The conflict matrix is

$$\underset{\sim}{C} = \begin{bmatrix} -b_x & -\alpha_y \\ -\alpha_x & -b_y \end{bmatrix} \qquad (25)$$

so that the system is stable when

$$\frac{\alpha_x}{b_y} \cdot \frac{\alpha_y}{b_x} < 1 \tag{26}$$

i.e., when the warfare efficiencies are relatively low. When efficiencies become too great, the system becomes unstable and diverges toward whichever side gains the initial advantage. There is no second equilibrium point. The regions defined by (26) are shown in Figure 6 for the $\left\{\alpha_x, \alpha_y\right\}$ parameter space.

Stability of Type 2 Systems

When we apply the Routh Test, (21), to the conflict matrix for Type 2 systems, (17), at unity equilibrium, we find that the system is environmentally stable at unity equilibrium since

$$-(b_x + b_y + \mu_x + \mu_y) < 0$$

and (27)

$$b_x\, b_y + \mu_x\, b_x + \mu_y\, b_y > 0$$

for all possible values of the parameters.

At equilibrium point two, which is always in the third quadrant, we find, with the aid of (11) that it is always unstable since

$$C_{11} + C_{22} = \mu_x\, b_x/b_y + \mu_y\, b_y/b_x > 0 \tag{28}$$

and the first condition of (21) is always violated. Furthermore

$$C_{12}\, C_{21} = \mu_x\, \mu_y\, X_e\, Y_e > 0 \tag{29}$$

so the solutions of the linear state equations are non-oscillatory.

To summarize; Type 2 systems, the counter-C^3 model (3c) and the model studied by Moose, [1980] for counter-C^3, are environmentally stable at unity equilibrium, and non-oscillatory. Their other equilibrium point which is in the third quadrant and is therefore unfeasible, is always unstable. Thus, we

may anticipate Type 2 systems to always tend back toward unity equilibrium, even from quite far away. In fact, they are probably globally stable at unity.

Stability of Type 3 Systems at Unity Equilibrium

At unity equilibrium, Type 3 systems are stable if

$$\left.\begin{aligned} &\mu_x(Y_o - 1) + \mu_y(X_o - 1) < b_x + b_y \\ \text{and} \quad &\left[\frac{b_x}{\mu_x} - (Y_o - 1)\right]\left[\frac{b_y}{\mu_y} - (X_o - 1)\right] < 1 \end{aligned}\right\} \tag{30}$$

Eq. (30) is quite similar to (23), the stability conditions at unity for Type 1a systems, but the second inequality is reversed. Figure 7 illustrates how the conditions of (30) generate a region of stability which is a diagonal strip in the $\left\{\mu_x, \mu_y\right\}$ parameter space. For either too small or too large values of μ_x and μ_y, the system is unstable.

We know that the second equilibrium point of Type 3 systems always lies in the third quadrant and is therefore not feasible. It, like unity equilibrium, is environmentally unstable. Its properties will also be described more fully in a subsequent report. However, we note here that

$$C_{12}\, C_{21} = \mu_x\, \mu_y\, X_e\, Y_e > 0 \tag{31}$$

holds for both points so that Type 3, like Type 2 systems, are non-oscillatory.

A summary of the principal system properties for each of the five models is presented below in Table 1.

	Type 1			Type 2	Type 3
	Mixed Attrition Eq. (2)	Counter-C^3 Eq. (3a)	Counter-C^3 Eq. (3b)	Counter-C^3 Eq. (3c)	Intelligence Eq. (4)
1) Unity Equilibrium Stability	E.U.	E.U.	E.U.	S.	E.U.
2) Equilibrium Point Two Locus	I,II,III,IV	I,II,III,IV	n/a	III	III
3) Equilibrium Point Two Stability	E.U.	E.U.	n/a	U.	E.U.
4) Oscillatory Solutions	yes (Equlibm 2)	yes (Equlibm 2)	no	no	no

Table 1

Summary of Principal System Properties

(E.U. = Environmentally Unstable, U = Always Unstable, S = Always Stable, State Space Quadrants I, II, III, IV.)

Discussion

The five models we have investigated in this paper are all generalized Lotka-Volterra equations for two-species systems. In spite of these restrictions, their interpretation in the C^3I context seems meaningful and the nature of their dynamic behavior quite remarkable.

The mixed attrition Lanchester type model with constant replenishment is quite useful to evaluate the increased effectiveness of forces as they become more precisely targeted against the enemies' forces. The interpretaton of the parameter μ_y for side Y can be: "μ_y increases from zero toward one as Y's targeting accuracy improves with better C^3I," or "μ_y decreases toward zero as counter-C^3 techniques by X confuse or decoy Y's aimed fire" or "μ_y is large if X, in order to attack, must expose his positions to Y who has accurate targetable weapons." (For example, mines are not targetable weapons but laser-aimed anti-tank missiles are.) All three of these interpretations for the μ parameter in the mixed attrition model are C^3I or counter-C^3 related. We also note that increasing μ tends to make the system unstable. One would suspect that the side with the greatest μ should win when instability occurs. Although this may happen, it is not guaranteed and in fact what does transpire depends critically on the location and stability of the second equilibrium point and on the initial conditions at the onset of the conflict. We will investigate this phenomenon in more detail in a separate report.

The three different models for information war involving counter-C^3 measures show the critical importance of our assumptions about how the counter-C^3 techniques actually effect the opposition's efforts to keep his information current and accurate. If, as was assumed in the previous paper [Moose, [1980]], these activities are effective against a well informed enemy, but

ineffective against an ignorant, or already confused one, then the dynamics are those of the Type 2 systems. These dynamics are remarkably simple and essentially independent of the actual values of the model parameters insofar as the gross system behavior is concerned. On the other hand, if counter-C^3 methods are in fact more effective against an enemy as he becomes more confused or ignorant, such as might occur through input overloading of his sensors and processing equipments, then the dynamics are those of Type 1a systems. These dynamics have two major modes, stable and unstable, and the mode of behavior is critically dependent on the actual model parameters.

Finally, in our intelligence information model, the behavior corresponds to Type 3 systems. Here again there are two modes of behavior but only one possible stable point in the feasible region. Therefore, either both sides are maintained with a fairly constant level of knowledge about the other or one side tends to know a great deal while the other knows very little. The first mode prevails when intelligence comes primarily from analysis of non-restricted or uncontrollable sources of information about the enemy. The second condition prevails when intelligence comes more predominately from breaks in the enemy's security; through espionage, communication intercepts, etc. This model too needs further examination as the stability of its second equilibrium point, which is in the third (non-feasible) quadrant, still has a great influence of the actual trajectory of this type of information war.

References

1. Brackney, H. 1959. The Dynamics of Military Combat. Opns. Res. 7, 30-44.

2. Intriligator, M.D., 1982. Research on Conflict Theory. J. of Conflict Resolution, V. 26 No. 2, 307-327.

3. Lanchester, F.W., 1916. Aircraft in Modern Warfare: The Dawn of the Fourth Arm. Constable and Co., London.

4. May, R.M., 1974. Stability and Complexity in Model Ecosystems. Princeton University Press, Princeton, NJ.

5. Moose, P.H., 1980. A Dynamic Model for C^3 Information Incorporating the Effects of Counter-C^3. NPS 62-81-025PR, Naval Postgraduate School, Monterey, CA.

6. Moose, P.H., 1983. Dynamics of Modern Military Combat, submitted to Operation Research. (also see: Moose, P.H., 1982. A Dynamic Model for Modern Military Combat, NPS 62-82-047, Naval Postgraduate School, Monterey, CA).

7. Richardson, L.F., 1960. Arms and Insecurity, The Boxwood Press, Pittsburgh, PA.

8. Truxal, J.G., 1972. Introductory Systems Engineering, McGraw-Hill, New York, NY.

FIGURE 1

Locus of Equilibrium Pt. Two
Mixed Attrition Type 1 Systems

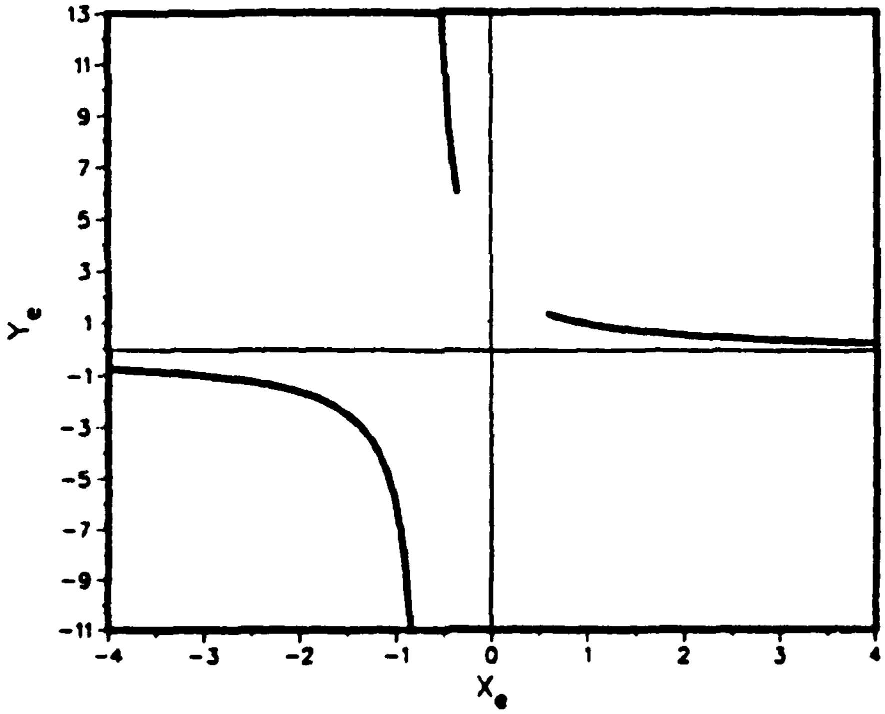


FIGURE 2

Locus of Equilibrium Pt. Two
Information War Type 1 Systems

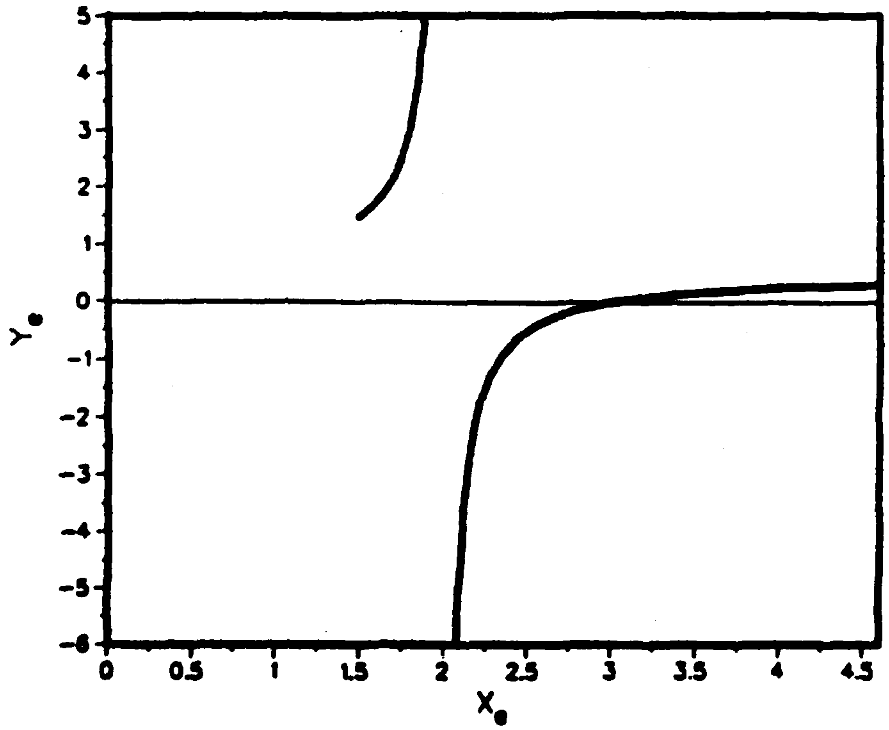


FIGURE 3

Stability Regions for Unity Equilibrium Mixed Attrition Type 1 Systems

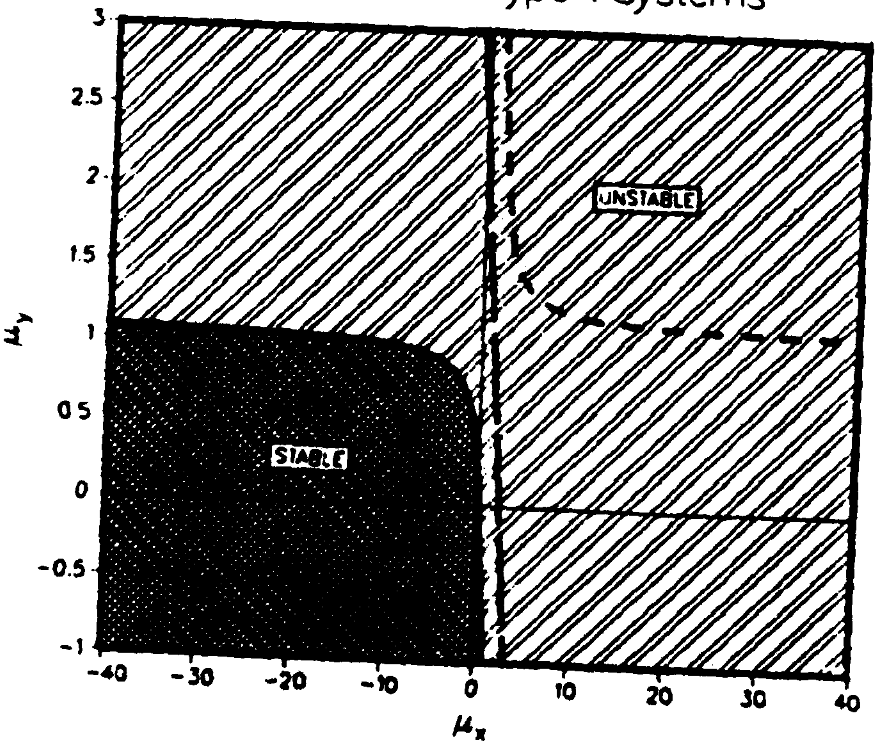


FIGURE 3a

Stability Regions for Unity Equilibrium Mixed Attrition Type 1 Systems

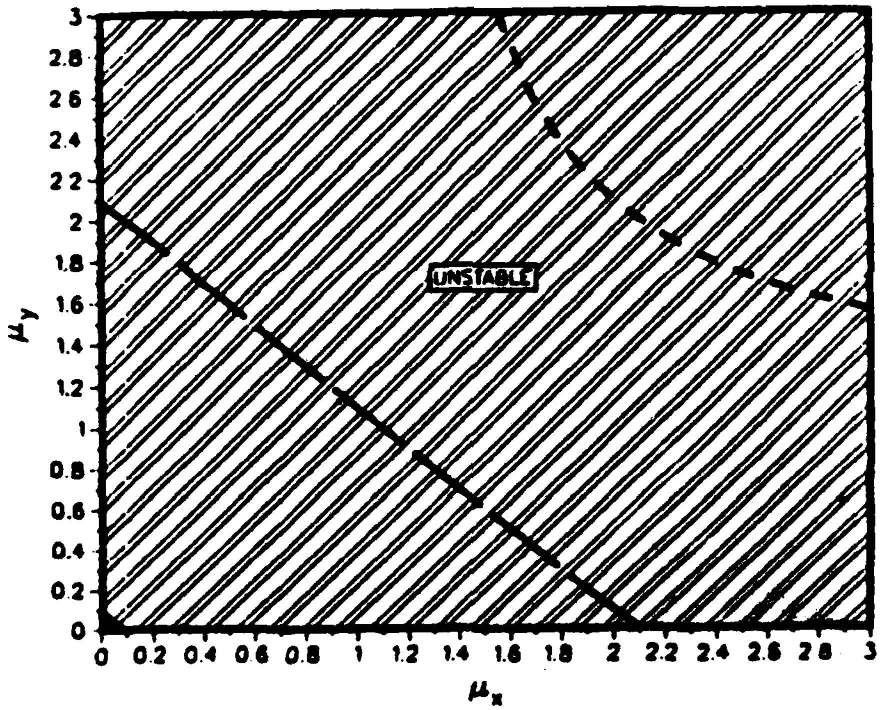


FIGURE 3b

Stability Regions for Unity Equilibrium Mixed Attrition Type 1 Systems

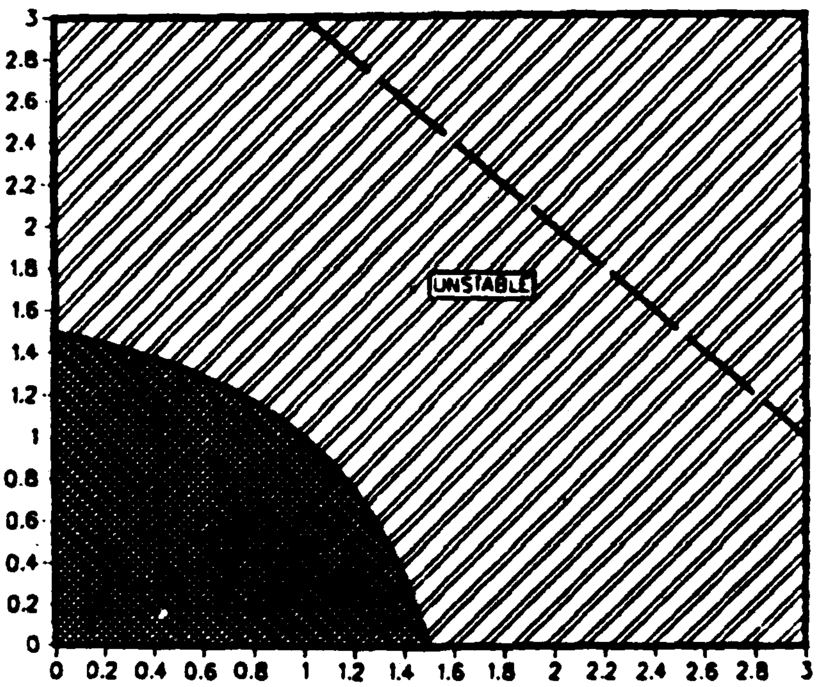


FIGURE 3c

Stability Regions for Unity Equilibrium
Mixed Attrition Type 1 Systems

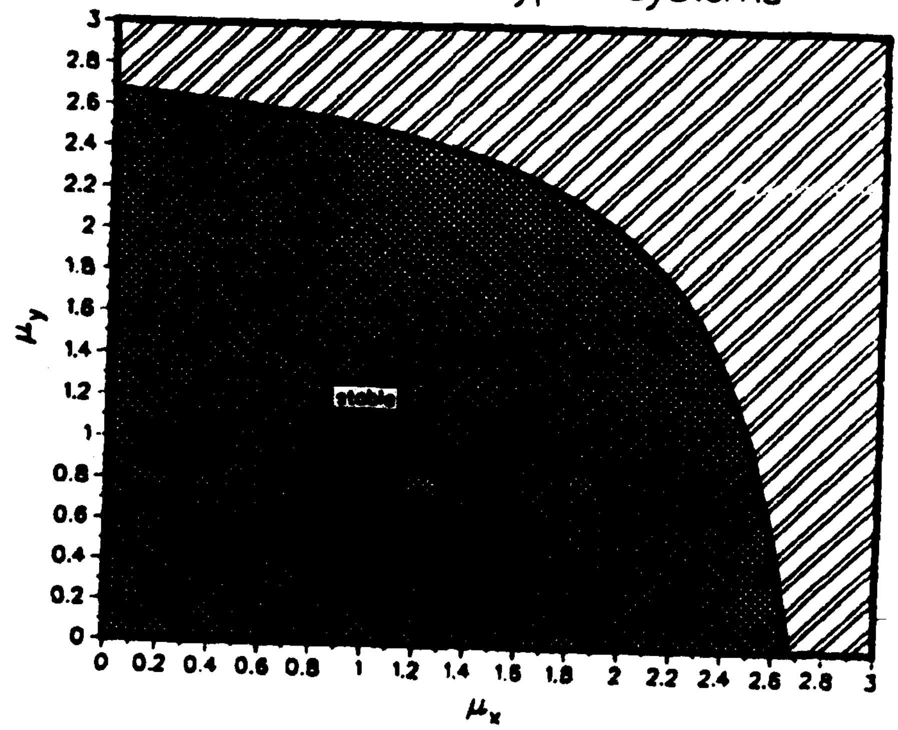


FIGURE 4a

Stability Regions for Unity Equilibrium
Information War Type 1 Systems

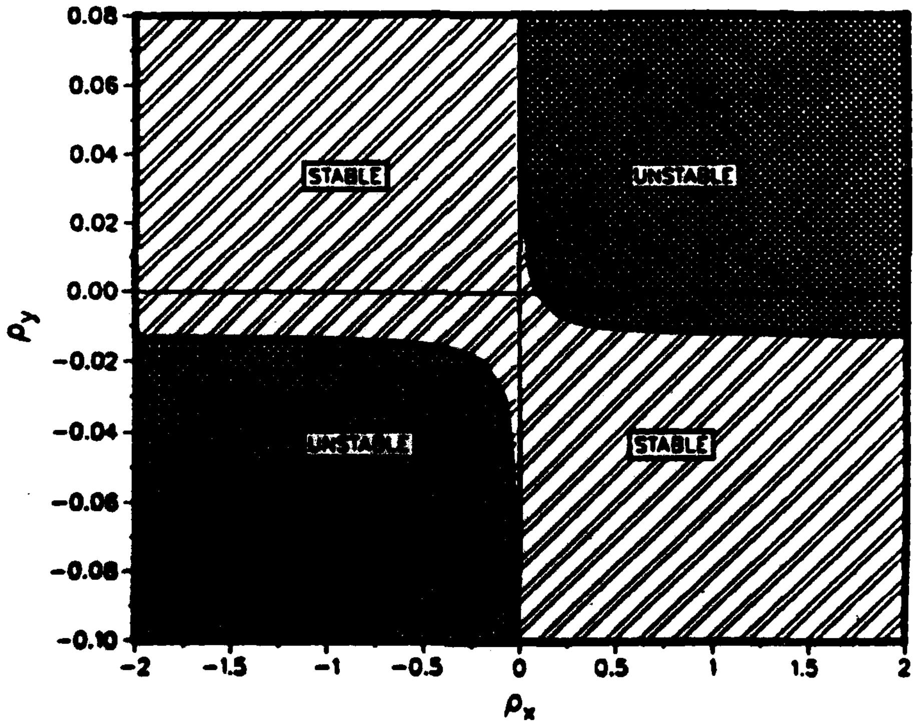


FIGURE 4b

Stability Regions for Unity Equilibrium Information War Type 1 Systems

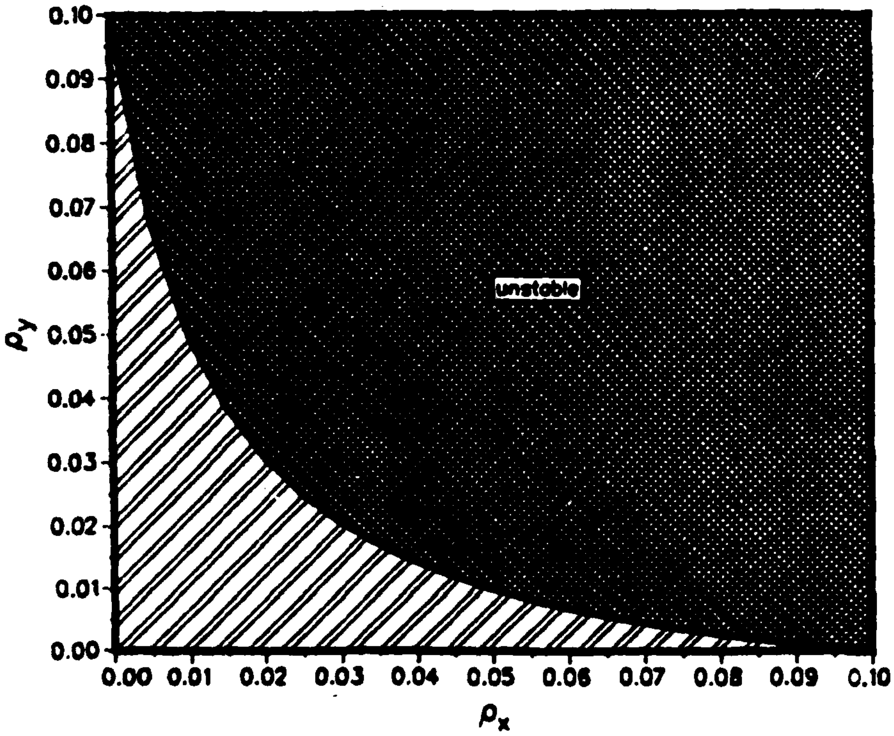


FIGURE 5a

Stable Phase Trajectory
Mixed Attrition Type 1 Systems

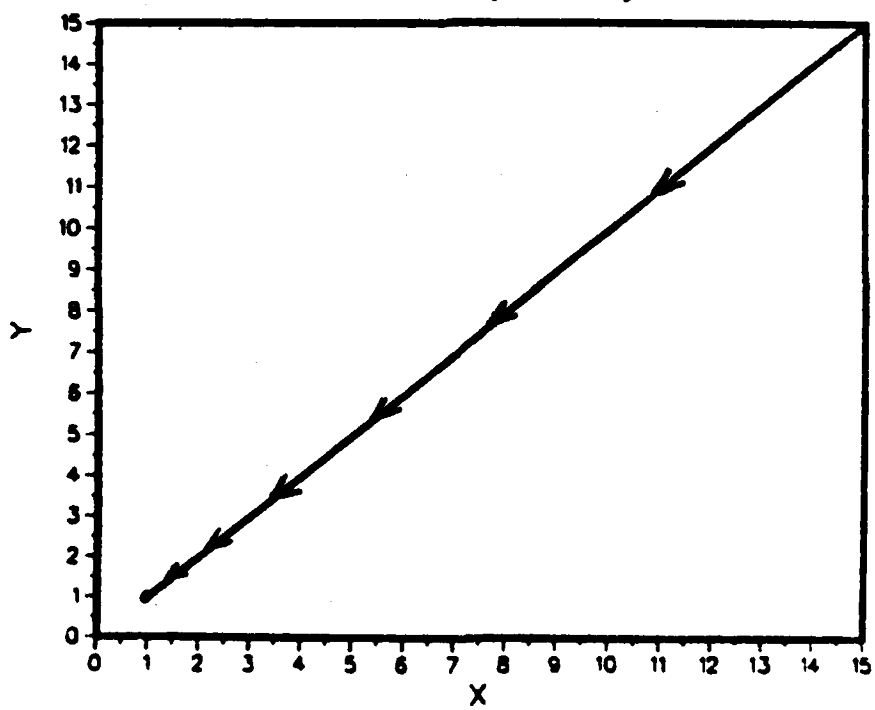


FIGURE 5b

Unstable Phase Trajectory
Mixed Attrition Type 1 Systems

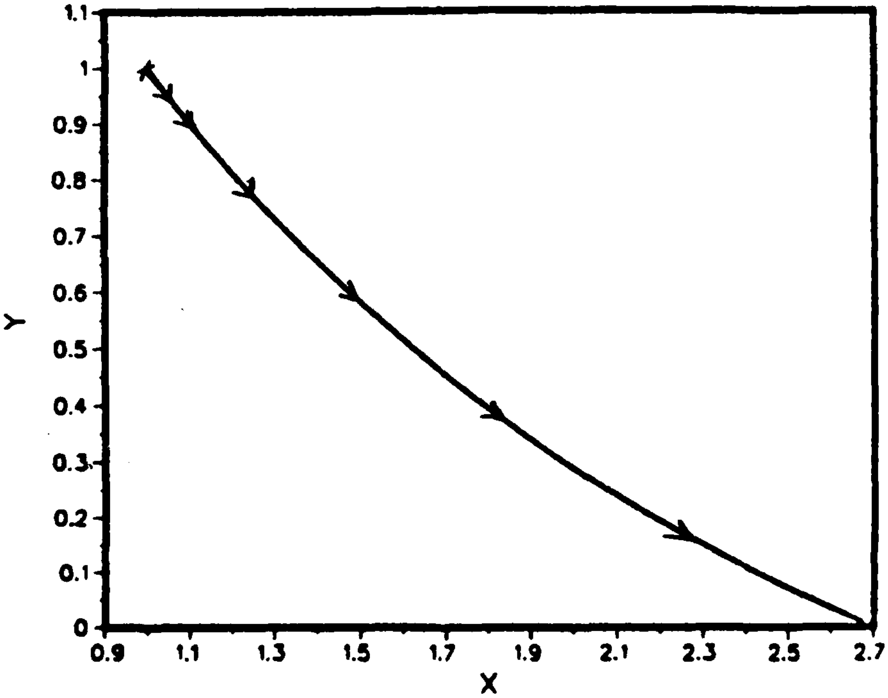


FIGURE 5c

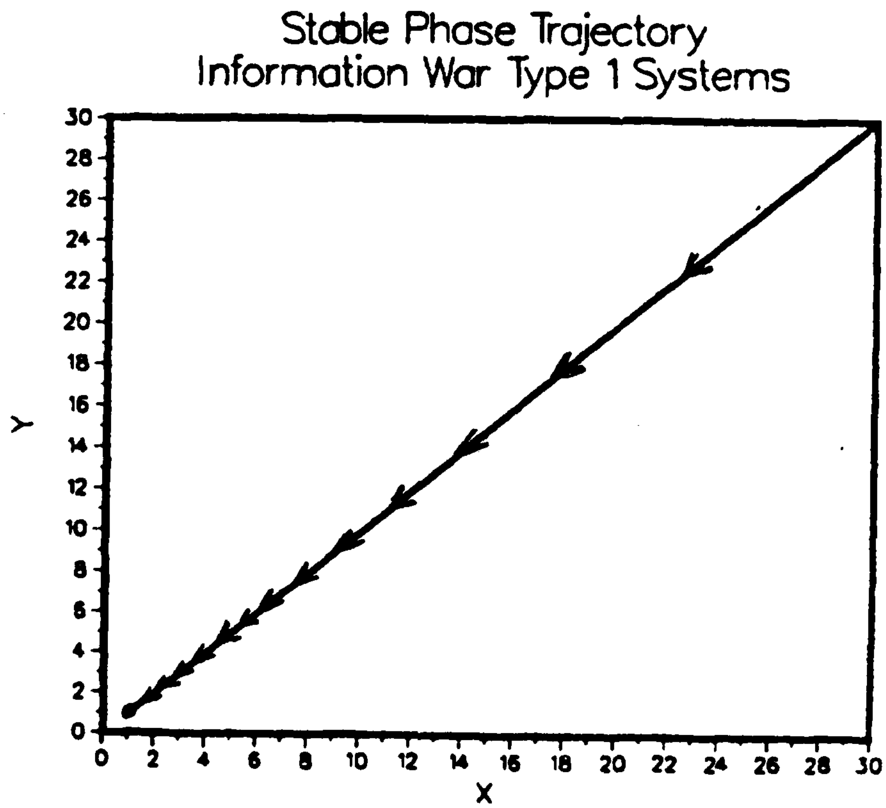

Stable Phase Trajectory
Information War Type 1 Systems
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
Y
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
X

FIGURE 5d

Unstable Phase Trajectory
Information War Type 1 Systems

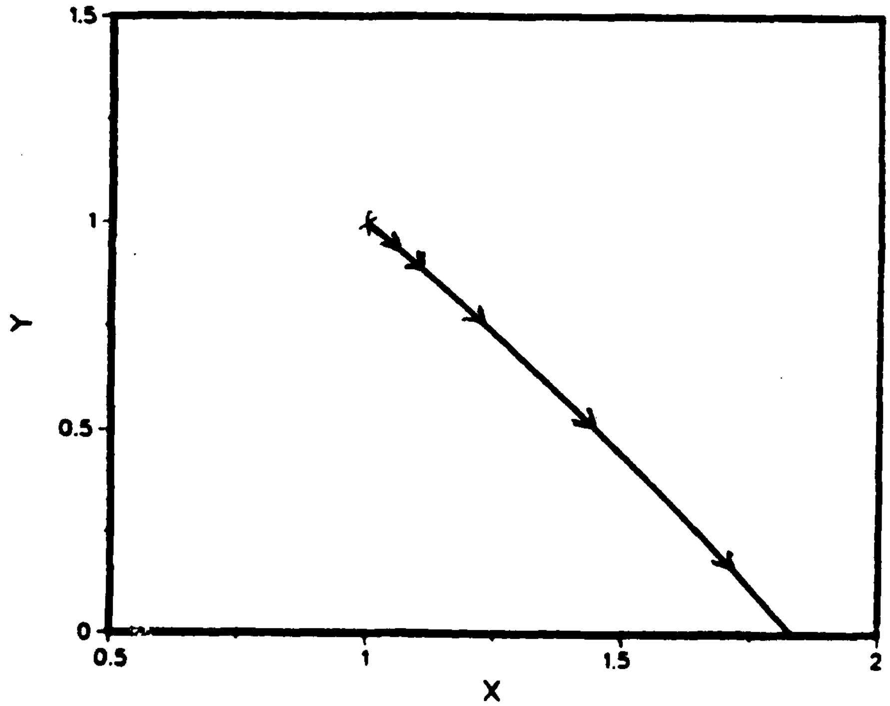


FIGURE 6

stability regions of linear coupled systems

$(\mu_x = \mu_y = 1)$

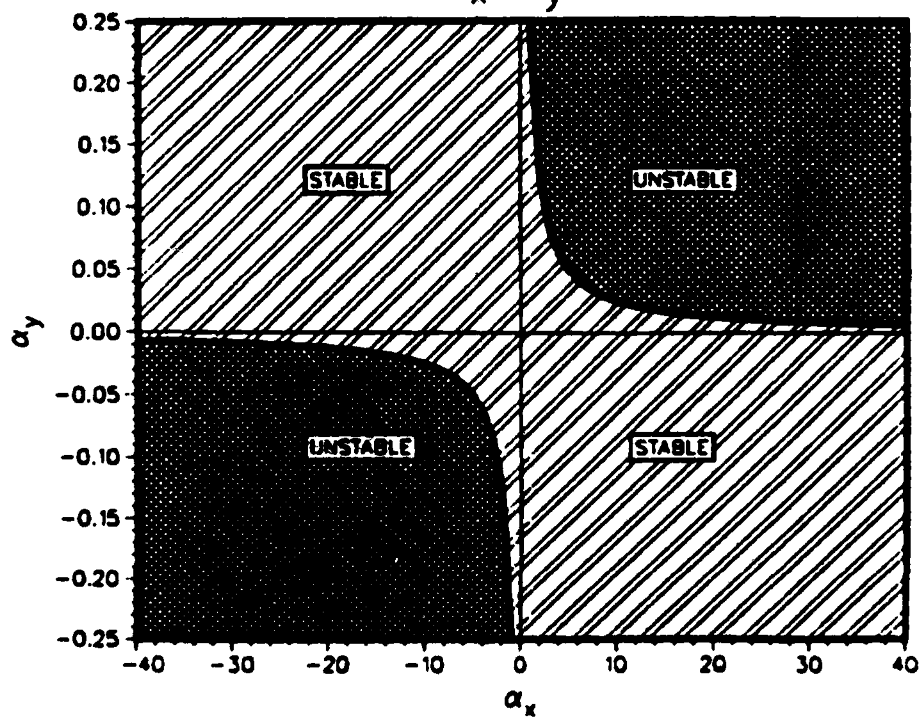
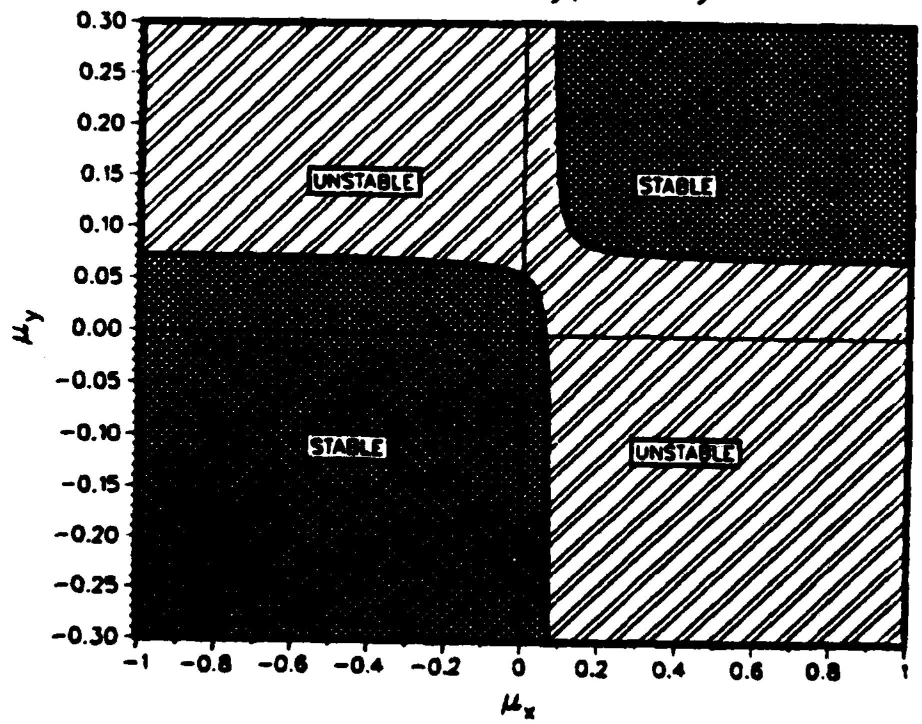


FIGURE 7

Stability Regions for Unity Equilibrium Information War Type 3 Systems



DISTRIBUTION

		No. Copies
1.	Library, Code 0212 Naval Postgraduate School Monterey, California 93943	4
2.	Dean of Research, Code 012 Naval Postgraduate School Monterey, California 93943	1
3.	Director of NET Assessment Attn: LtCol Fred Giessler Office of Secretary of Defense Room 3A930, Pentagon Washington, DC 20301	1
4.	Dr. Joel Lawson Technical Director Naval Electronics Systems Command Department of the Navy Washington, DC 20360	1
5.	Office of Chief of Naval Operations Attn: Dr. Daniel Schutzer, OP 009T The Pentagon Washington, DC 20301	1
6.	Professor K.E. Woehler, Code 61Wh Department of Physics & Chemistry Naval Postgraduate School Monterey, California 93943	1
7.	Professor Paul H. Moose, Code 62Me Department of Electrical Engineering Naval Postgraduate School Monterey, California 93943	25
8.	Dr. Thomas P. Rona Boeing Aerospace Co. P.O. Box 3999 Seattle, Washington 98124	1
9.	Office of Naval Research Attn: Dr. Charles Holand Arlington, Virginia 22217	1
10.	Laboratory for Information and Decision Systems MIT Attn: Dr. Robert R. Tenney Cambridge, Massachusetts 02139	1

DATE
ILMED
8